

Game Theory Take-Home Exam

Spencer Stamats

3.

a) **It is the final few hours of the Battle of Waterloo, and Napoleon must decide what to do.** All day he has battered the British lines—managing to take their three improvised fortresses—but the British still hold the ridge where they’ve contested the French advance. The Prussians are fast approaching, trying to rejoin with their British allies, and have already begun to skirmish with French units. Almost all the cavalry is dead due to foolish charges earlier in the battle, leaving Napoleon with very few options. He can charge the British center, and hope to break them with the sheer weight of numbers, perhaps committing his famed Imperial Guard regiment to the fray. However, Wellington is known for being able to hold a defensive position, so if the British stay stationary, the odds may be against Napoleon. He can attempt to flank the British lines, gaining a position on their ridge and driving them off of it; however, this leaves his center weak, and if the British decide to push forwards, they may be able to recapture their farmhouse-fortresses. Third, he can wait and see what the British do, and if they do nothing, withdraw from the field and fight the next day against the combined might of both Great Britain and the Prussians—a battle which he may be able to win, but will be incredibly difficult.

b)

		Wellington	
		Push	Wait
Napoleon	Attack	85%	40%
	Flank	30%	95%
	Wait	65%	25%

The percentages are the chances of Napoleon winning, and otherwise he loses. If Napoleon attacks and Wellington pushes, Napoleon has a good chance of winning, while if he attacks and Wellington waits, then the odds tilt in Wellington’s favor. If Napoleon flanks and Wellington waits, Napoleon is near certain of winning, but if Wellington pushes forwards, then he’ll take his strong-points back and likely be able to hold until the Prussians led by Blucher arrive. If Napoleon waits and the Wellington pushes, then Napoleon will probably be able to defeat him on the field of battle, but it is less probably than if he were on the offensive. If they both wait, then night will fall, and on the next day Napoleon will be facing both Wellington and Blucher, and stands very little chance of winning at all. These percentages are of course approximations, but come close to reflecting the relative odds in the situation.

c) This game is a zero-sum game. For every percentage point of odds of winning that Napoleon gains, Wellington must lose. There are only 100 percentage points to distribute, leading to a finite amount of utility (as victory is the only thing of import to these generals), and an overall conservation of it.

For Napoleon, the strategy of waiting is dominated by that of attacking. In both situations attacking would serve him better: if Wellington attacks, the odds are better of beating him, and if he attacks the odds are still against Napoleon, but he has a much better chance against Wellington on a hill than Wellington and Blucher together. So waiting is a dominated strategy.

Utility, in this game, is almost a non-issue. The only thing of interest to these generals is victory. The fact that assaulting up a hill would cause more casualties doesn't factor into Napoleon's decision, nor does the glory of a charge into Wellington's. Any other utility, such as the value of the fortress/strong-points, or of a hill, or flanking maneuver, is already taken into account into the odds of victory.

Both generals are rational actors. They will do what it takes to win the battle, and will not do something such as charge for irrational reasons—glory, fame, etc.

For Napoleon, the best-worst case strategy is to attack. In that situation, no matter how Wellington reacts, he still has a 40% chance of winning, compared to 30% and 25%.

The concept of expected value doesn't have much impact on this decision. The expected value (using the percentage values I came up with) is the same for both of the non-dominated strategies. In the end, this decision is based largely off of what Napoleon knows about his opponent. Historically, Wellington was a more defensive general, and so would favor the strategy of waiting. However, Napoleon did not know this, and so chose to charge the hill, relying on his crack Imperial Guard regiment to break the British lines.

Historical Note: There are definitely inaccuracies in this portrayal of the Battle of Waterloo. One of the largest is that the British were battered and near beaten, and would not have considered a charge of any sort. Napoleon did not choose to attack up the hill because there was any chance of the British meeting him, but because he didn't understand the effectiveness of the British line against the French column, having never faced British troops before. His Imperial Guardsmen were beaten, and when they ran they took the rest of the army with them. But really, the choices were not structured as I showed them to be here; I heavily adapted the situation to fit a more game theory-esque model.